CONVECTIVE INSTABILITY OF A LAYER OF VISCOELASTIC FLUID TAKING THE THERMAL CONDUCTIVITY OF THE BOUNDARIES INTO ACCOUNT

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INTRODUCTION

It was assumed in [1], when investigating the equilibrium stability of a viscoelastic fluid, that temperature perturbation vanishes at the planes bounding the layer. These conditions correspond to the limiting case of boundaries with an infinite thermal conductivity. In the case where the dividing boundaries have a finite thermal conductivity, the penetration of temperature perturbations into the body of the fluid must be taken into account. The present paper considers the stability problem of a layer of viscoelastic fluid confined between two semiinfinite masses of finite thermal conductivity.

In the general case of a viscoelastic fluid the stress tensor at a point in the medium is given as a functional of the history of the strain [2]. Particular rheological models can be obtained by specifying the form of the functional kernels.

Methods for measuring the material constants appearing in the rheological equations are complicated and require complex rheological devices.

Information obtained from viscometric flow cannot be used for nonviscometric flow [3].

In the latter case the number of material functions appearing in the defining relations [4] increases. Thus, in each particular case the contribution from additional functionals describing all the interactions allowed by the appropriate symmetry must be estimated for the case of nonviscometric flow.

Assuming that the memory of the fluid decays fairly rapidly, and that the rate of strain is negligible, we can represent the stress tensor for the case treated in this paper in the form of the integral model discussed previously in [1].

We shall consider a horizontal layer of viscoelastic fluid heated from below which is bounded by the planes $z=0$ and $z=d$. The layer is bounded by semiinfinite solid masses whose thermal conductivities $\varkappa_1$ and $\varkappa_2$ are different from each other and from the thermal conductivity $\varkappa$ of the fluid (see Fig. 1).

In the Boussinesq approximation the amplitude equations for perturbations in the fluid are written as follows:

$$\{(D^2-a^2)[\psi(\sigma)\mathrm{Pr}(D^2-a^2)-\sigma]\}W=\mathrm{PrRa}a^2\Theta; \tag{1}$$

$$(D^2-a^2-\sigma)\Theta=-W. \tag{2}$$

The following equation is valid for the temperature perturbations in the masses:

$$(D^2-a^2-\sigma\bar{\varkappa}_s^{-1})\Theta_s=0\ (s=1,2), \tag{3}$$

where W and Θ are the amplitudes of velocity and temperature perturbations in the fluid; $\bar{\varkappa}_S=\varkappa_S/\varkappa$; $\varkappa_S$ is the thermal conductivity of the boundaries; Pr, Ra are the Prandtl and Rayleigh numbers; a is the wave number; σ is the perturbation decrement; $\psi(\sigma)=\eta_0^{-1}\int_0^\infty N(\tau)(1+\sigma\tau)^{-1}d\tau$, $N(\tau)$ is the distribution function of relaxation times; η_0 is the largest Newtonian viscosity; and $D=\partial/\partial z$. Here the variables have been made dimensionless by division by the following quantities; the distance is divided by the layer thickness d, the time by d^2/χ, the velocity by χ/d, and the temperature by βd. Here χ is the thermal diffusivity of the fluid, $\beta=(T_2-T_1)/d$.

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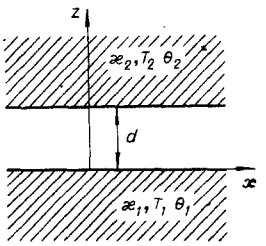

Fig. 1

TABLE 1

Pr	λ	$\bar{\varkappa}$	$\bar{\varkappa}_1=\infty$, $\bar{\varkappa}_2=\bar{\varkappa}$			$\bar{\varkappa}_1=0$, $\bar{\varkappa}_2=\bar{\varkappa}$			$\bar{\varkappa}_1=\bar{\varkappa}_2=\bar{\varkappa}$		
			Ra_*	a_*	ω_*	Ra_*	a_*	ω_*	Ra_*	a_*	ω_*
10	0,1	∞	(230,0)	(7,3)	(76,6)						
		∞	228,7	6,7	71,32	231,3	7,0	73,8	228,7	6,7	71,3
		2,00	228,9	7,0	73,91	232,5	7,0	73,7	231,4	6,5	69,4
		0,20	229,6	7,3	76,55	234,1	7,0	73,6	232,6	71	74,6
		0,02	231.1	7,4	77,42	235,0	7,0	73,65	233,4	7,0	73,8
	1,0	∞	(7,49)	(4,72)	(20,77)						
		∞	7,08	4,9	21,15	7,23	5,4	22,1	7,08	4,9	21,15
		2,00	7.01	4,6	20,65	7,15	4,6	20,6	7,01	4,8	20,95
		0,20	7.14	5,0	21,3	7,28	4,8	20,9	7,31	5,0	21,25
		0,02	7.10	5,1	21,3	7,24	4,8	20,9	7,27	4,8	20,9
		0,00	6,84	5,0	21,3	6,84	5,0	21,3	6,84	5,0	21,3
100	0,1	∞	(130,1)	(11,96)	(385,8)						
		∞	129,5	11,94	381,2	135,0	8,4	276,1	129,5	11,9	381,2
		2,00	133,4	8,3	272,5	135,1	8,4	276,0	130,0	8,4	276,0
		0,20	134,5	8,3	272,1	135,3	8,3	273,2	133,1	8,4	276,0
		0,02	135,5	8,3	271,0	135,6	8,3	273,0	135,0	8,4	276,1
	1,0	∞	(2,203)	(7,29)	(83,45)						
		∞	2.201	7,29	83,51	2,08	6,4	76,6	2,20	7,29	83,51
		2,00	2,032	6,31	75,91	2,09	6,4	76,6	2,08	6,41	76,6
		0,20	2,037	6,30	75,9	2,09	6,4	76,6	2,09	6,40	76,6
		0,02	2,038	6,3	75,9	2,03	6,0	73,8	2,09	6,40	76,6

The boundary conditions for amplitude perturbations, appropriate for the present problem, are

$$\begin{gathered}\Theta = \Theta_1 \varkappa_1 \; D\Theta_1 = \varkappa D\Theta \quad \text{for} \quad z = 0;\\ \Theta = \Theta_2, \; \varkappa_2 D\Theta_2 = \varkappa D\Theta \quad \text{for} \quad z = 1;\\ W = DW = 0 \quad \text{for} \quad z = 0,1;\\ \Theta_1 = \Theta_2 \to 0 \quad \text{for} \quad z \to \pm\infty.\end{gathered} \tag{4}$$

The spectrum of decrements σ and of the corresponding characteristic perturbations can be determined from the homogeneous boundary problem (1)-(4). Monotonic or oscillatory instability arises depending on the properties of the fluid. It has been established that for a Newtonian fluid the "principle of monotonic perturbations" is valid and that the stability threshold is defined by the value where the decrement becomes zero [5, 6]. The presence of elasticity in non-Newtonian viscoelastic fluids is an additional destabilizing factor, and the principle in question is violated [1].

Galerkin's method is applied to solve the problem. Following [7] we adopt an approximation of the form

$$W = qz^2(1 - z)^2, \tag{5}$$

where the coefficient q is chosen from the normalizing condition and is set equal to unity in view of the homogeneity of the problem. We now use Eq. (5) to determine the temperature perturbation in the fluid layer from Eq. (2):

$$\Theta = C_1 \operatorname{sh} kz + C_2 \operatorname{ch} kz + z^4/k^2 - (2/k^2)z^3 + [(12 + k^2)/k^4]z^2 - (12/k^4)z + 2(12 + k^2)/k^6,$$

where $k^2 = a^2 + \sigma$.

When Eq. (4) is taken into account, the solution of Eq. (3) can be represented in the form

$$\Theta_1 = A \exp(\gamma_1 z), \quad \Theta_2 = B \exp[\gamma_2(1-z)],$$

where $\gamma_s = \sqrt{a^2 + o/\bar{\varkappa}_s}$.

The constants C_1, C_2, A, and B are determined from the boundary conditions for temperature (4). The expressions for C_1 and C_2, necessary for determining the critical Rayleigh number, turn out to be fairly cumbersome and difficult to review and so are not given here. For the particular case of the absence of oscillatory instability, when the substitution $\bar{\varkappa}_S = 1/\varkappa_S$ is made, the values of C_1 and C_2, as well as the magnitude of the critical Rayleigh number, coincide completely with the data of [7].

The integral condition of Galerkin's method leads to the function for the appearance of oscillatory instability. When the neutral state is characterized by $\sigma = i\omega$ the expression is the complex viscosity. The value of the Rayleigh number is

$$\mathrm{Ra} = \frac{D_1 D_3 + D_2 D_4}{D_3^2 + D_4^2} + i\,\frac{D_2 D_3 - D_1 D_4}{D_3^2 + D_4^2}, \tag{6}$$

where

$$D_1 = (\mathrm{Pr}\eta'/630\eta_0)(504 + 24a^2 + a^4);$$
$$D_2 = (1/630\eta_0)[\omega\eta_0(12 + a^2) - \mathrm{Pr}\eta''(a^4 + 24a^2 + 504)];$$
$$D_3 = \mathrm{Pr}a^2 M/630(a^4 + \omega^2)^3; \quad D_4 = \mathrm{Pr}a^2 N/630(a^4 + \omega^2)^3;$$
$$M = M(a, \omega, C_1, C_2, p, q); \quad N = N(a, \omega, C_1, C_2, p, q).$$

Setting the imaginary part of Eq. (6) equal to zero defines the wave number as a function of the oscillation frequency ω:

$$p = \mathrm{Re}[(a^2 + i\omega)^{1/2}], \quad q = Im[(a^2 + i\omega)^{1/2}].$$

Equation (6) gives the value of the critical Rayleigh number as a function of the following parameters: the wave number a, the ratios of the thermal conductivities of the masses $\varkappa_S$, the elasticity parameter, the oscillation frequency, and the Prandtl number. The critical Rayleigh number was determined on a model "M-222" computer for the case when the following relations are valid for the components of the complex viscosity:

$$\eta' = \eta_0/[1 + (\bar{\omega}\bar{\lambda})^2], \quad \eta'' = \eta_0\bar{\omega}\bar{\lambda}/[1 + (\bar{\omega}\bar{\lambda})^2],$$

where $\bar{\lambda}$ is the Maxwell relaxation time. The parameter $\lambda = \bar{\lambda}\chi/d^2$, a measure of the elasticity, is the ratio of the relaxation time of the stresses to the thermal relaxation time. In the general case the complex viscosity is determined from dynamic experiments as a function of the frequency. The dimensionless frequency has the form $\omega = \bar{\omega}d^2/\chi$, and so the quantity d^2/χ should be determined for real fluids.

Those values of ω were chosen from the set of roots of the equation $a = a(\omega)$ which gave the least real value of Ra.

Values of the calculated critical parameters Ra_*, a_*, ω_*, corresponding to loss of stability, are given in Table 1, from which it can be concluded that the thermal conductivity of the boundaries has a negligible effect on the appearance of oscillatory instability (values of the critical parameters from [8] are given in the brackets). For a Newtonian fluid the stability is monotonically reduced as the thermal conductivity of the bounding masses decreases, while the wavelength of critical perturbations increases [7].

Oscillatory instability is encountered in fluids where the ratio of the relaxation time to the thermal relaxation time is large [1]. The penetration of temperature perturbations into the masses increases the effective dimensions of the regions where perturbations exist. The relatively greater freedom for the development of temperature perturbations leads to a decrease in the relaxation time of the fluid, and inhibits the appearance of oscillatory instability, while the presence of boundaries with finite thermal conductivity leads to a reduction in stability [5].

The joint occurrence of these two phenomena leads to the result that the thermal conductivity of the boundaries exerts no appreciable effect on the appearance of oscillatory instability. If the relaxation time decreases sufficiently for the fluid to be regarded as having a very short memory, then instability arises through the stationary state.

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GENERAL THEORY OF HEAT AND MASS EXCHANGE IN CHEMICALLY REACTIVE SYSTEMS IN MECHANICAL EQUILIBRIUM WITH ELECTRIC FIELD WITHIN THE FRAMEWORK OF THERMODYNAMICS OF IRREVERSIBLE PROCESSES

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General analysis of the effects of chemical reactions on the processes of heat and mass transfer has been the subject of many investigations (see literature cited in [1]). Such an analysis for mechanical equilibrium systems in symmetrical form was given in [2]. In the present article it is shown that a description of chemical processes in the special meaning of the term can be carried out independently of heat and mass transfer.

The processes which take place in a mechanical equilibrium (at rest) system consisting of k chemical components K_i(i, j = 1, ..., k), among which r independent reactions R_s(s, t = 1, ..., r) occur, are described by k continuity equations,

$$\rho \frac{\partial c_i}{\partial t} + \operatorname{div} \mathbf{I}_i = \sum_s m_i \nu_{is} \theta_s, \tag{1}$$

where, in addition to other notation, c_i is mass fraction; $\mathbf{I}_i$ is diffusion flow; m_i is molecular weight, g/mole; θ_s is the rate of R_s, mole/cm^3·sec; ν_{is} is stoichiometric coefficient of K_i in R_s. Moreover, we have the mechanical equilibrium condition

$$\nabla p = \rho z \mathbf{E}, \tag{2}$$

where z is the free charge per unit of mass, as well as the energy equation which in usual notation is given by

$$\partial \rho \varepsilon / \partial t + \operatorname{div} \mathbf{q} = (\mathbf{jE}). \tag{3}$$

The system (1)-(3) is supplemented by the Gibbs relation,

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